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Theoretical analysis of thermal damages in skin tissue induced by intense moving heat source

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ABSTRACT

In this paper, the analytical solution of hyperbolic bio-heat equation under intense moving heat source is presented. The exact solution in the domain of Laplace's transform is obtained. The thermal damages to the tissues are evaluated by the extent of the denatured protein employing with the Arrhenius equation. The results indicate that the hyperbolic bio-heat model reduces to the parabolic bio-heat model when the thermal relaxation time is zero. Numerical results for temperatures and thermal damages are represented graphically. The effects of heat source velocity on the temperature of skin tissue and thermal damages are studied. These results can be used as a confirmation part for studying the practical operations such as scanning laser treatment and other numerical solutions.

1. Introduction

In the recent years, the temperature estimation in living tissue is under the concentration of the researchers. The thermal treatment method has been widely used in modern medicine, such as laser surgery [1], infrared irradiation [2], laser tissue soldering [3], hyperthermia [4] and other therapy methods. Between these clinical processes, the moving heat sources applications to living tissue considering exudation rate is observed in certain plastic surgery operation such as the removal of moles or laser tattoos or the thermal action of the cornea to correct hyperopia. Heat transfer in skin tissues are complex operations that contains heat conduction in the tissues and vascular system, convection between tissues and blood cause to blood flow, perfusion into canicular tissues, metabolic heat, sweating, etc. In 1948, Pennes [5] established the parabolic bio-heat equation, in which blood perfusion and metabolic thermal production are imported into the equation as a convective term. There are several numerical and analytical solutions for this equation in the literature. For instance, Gupta et al. [6] utilized the electromagnetic radiation to investigate the thermal therapy by using the finite element method. Dillenseger and Esneault [7] studied the development of temperature over time in hypothermia by using finite difference method. Zhu et al. [8] used the theory of diffusion to estimate the deposition of light energy in tissues and the rate process model for the resulting thermal damages. Diaz et al. [9] used the finite element scheme to get the solution of heat diffusion model in the tissues to improve the thermal damages models for laser irradiated cartilages. The analytical solutions are very interested due to their exact estimation and lower cost in comparison with experiment and numerical calculations. There are some analytical solutions for this problem too. Brix et al. [10] introduced an analytical solution for the Penne's bioheat model using the Green's function for investigating thermal response to Radio Frequency heating. Ahmadikia et al. [11] presented the solutions of parabolic and hyperbolic bio-heat equations analytically in the skin tissues induced by constant and transient heat flux. Kengne and Lakhssassi [12] solved analytically a simplified one-dimensional spherical form of bioheat model, using the separation of variable method combined with the Green's function method. Rodrigues et al. [13] derived an exact solution for one-dimensional form of bioheat model in cylindrical or spherical coordinates, considering multi-layer region. There are several phenomena such as moving laser processing in welding or laser alloying, which can be modeled as moving spot heating source. Abbas [14–17] studied some thermoelastic problems due to moving heat source. In addition, Marin [18,19] investigated the thermoelastic interactions in porous media. To our knowledge, there is no analytical solution for the hyperbolic bio-heat model under a moving heat source.

In this paper, the analytical solution for the hyperbolic bio-heat model under a moving heat source is introduced. Such as the interaction of the continuous scanning laser, the numerical results can

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be used as a confirmation division for the interaction of living tissue subjected to moving heat source.

2. Statement of the problem

The geometric model of skin tissues is established as Fig. 1. By considering the notion of finite thermal propagation speed. Based on Cattaneo [20] for heat flux inclusive the characteristic time τ_o as well as the Pennes's model, a general form of the heat wave pattern of bio-heat transfer in skin tissue is established by [11,21]:

$$k\nabla^2 T = \left(1+\tau_o\frac{\partial}{\partial t}\right)\left(\rho c\frac{\partial T}{\partial t}+\omega_b\rho_b c_b(T-T_b)-Q_m-Q_{ext}\right), \tag{1}$$

where is the tissue mass density, ρ_b is the blood mass density, T is the tissues temperature, τ_o is the thermal relaxation time, T_b is the blood temperature k is the tissue thermal conductivity, c is the specific heat of tissue, ω_b is the blood perfusion rate, Q_m is the metabolic heat generation in skin tissues, c_b is the blood specific heat, t is the time and Q_{ext} is the moving line heat source. We consider a finite domain of skin tissue with a thickness L with its surface and its bottom boundary are assumed to be thermally insulated as in Fig. 1.

So that the models of bio-heat equation with external heat source have the following form [11]

$$k\frac{\partial^2 T}{\partial x^2} = \left(1+\tau_o\frac{\partial}{\partial t}\right)\left(\rho c\frac{\partial T}{\partial t}+\omega_b\rho_b c_b(T-T_b)-Q_m-Q_{ext}\right). \tag{2}$$

In order to solve the equation of bio-heat conduction, two initial conditions following the description of the physical model are necessary:

$$T(x,0)=T_b,\quad \left.\frac{\partial T(x,t)}{\partial t}\right|_{t=0}=0.0. \tag{3}$$

In the bio-heat transfer models under consideration, both upper surface and the lower boundary are assumed to be thermally insulated.

$$-k\left.\frac{\partial T(x,t)}{\partial x}\right|_{x=0}=0,\quad -k\left.\frac{\partial T(x,t)}{\partial x}\right|_{x=L}=0. \tag{4}$$

For convenience, the non-dimensional quantities can be introduced by

$$(t',\tau_o')=\frac{\omega_b\rho_b c_b}{\rho c}(t,\tau_o),\quad \mathrm{T}'=\frac{\mathrm{T}-\mathrm{T}_b}{\mathrm{T}_b},$$
$$x'=\sqrt{\frac{\omega_b\rho_b c_b}{k}}x,\quad (Q_m',Q_{ext}')=\frac{(Q_m,Q_{ext})}{\omega_b\rho_b c_b T_b}. \tag{5}$$

In terms of these dimensionless form of variables in (5), Eqs. (2)–(4) can be written in the following forms (the prime has been dropped for convenience)

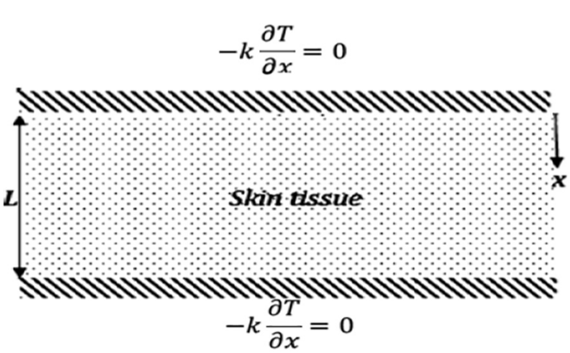

Fig. 1. Schematic diagram of biological tissue.

$$\frac{\partial^2 T}{\partial x^2} = \left(1+\tau_o\frac{\partial}{\partial t}\right)\left(\frac{\partial T}{\partial t}+T-Q_m-Q_{ext}\right), \tag{6}$$

$$T(x,0)=0,\quad \left.\frac{\partial T(x,t)}{\partial t}\right|_{t=0}=0, \tag{7}$$

$$\left.\frac{\partial T(x,t)}{\partial x}\right|_{x=0}=0,\quad \left.\frac{\partial T(x,t)}{\partial x}\right|_{x=L}=0. \tag{8}$$

The non-dimensional source term $Q_{ext}(x,t)$ is a moving line heat source which can be expressed as [14]:

$$Q_{ext}(x,t)=Q_o\delta(x-vt), \tag{9}$$

where v is constant velocity, δ is the delta function and Q_o is constant.

3. Laplace's transforms

The definition of Laplace transforms for every function $\Omega(x,t)$ by

$$\bar{\Omega}(x,s)=L[\Omega(x,t)]=\int_0^\infty \Omega(x,t)e^{-st}dt,\ s>0, \tag{10}$$

where s is the parameter of Laplace's transforms. Hence, the above equations can be given by

$$\frac{d^2\bar{T}}{dx^2}=(1+s)(1+s\tau_o)\bar{T}-\frac{Q_m}{s}-(1+s\tau_o)\frac{Q_o}{v}e^{-\frac{sx}{v}}, \tag{11}$$

$$\left.\frac{d\bar{T}(x,s)}{dx}\right|_{x=0}=0.0,\quad \left.\frac{d\bar{T}(x,s)}{dx}\right|_{x=L}=0.0. \tag{12}$$

By using the boundary conditions (12), The general solution of inhomogeneous Eq. (11) can be written in the form

$$\bar{T}(x,s)=A_1e^{-\alpha x}+A_2e^{\alpha x}+\frac{\beta}{\alpha^2}-\frac{\epsilon}{\zeta^2-\alpha^2}e^{-\zeta x}, \tag{13}$$

where $\alpha^2=(1+s)(1+s\tau_o)$, $\beta=\frac{Q_m}{s}$, $\epsilon=(1+s\tau_o)\frac{Q_o}{v}$, $\zeta=\frac{s}{v}$, $A_1=\frac{e^{-L\zeta}(-e^{L\alpha}+e^{L\zeta})\epsilon\zeta}{(-1+e^{2L\alpha})\alpha(-\alpha^2+\zeta^2)}$ and $A_2=-\frac{e^{L\alpha-L\zeta}(-1+e^{L\alpha+L\zeta})\epsilon\zeta}{(-1+e^{2L\alpha})\alpha(\alpha^2-\zeta^2)}$.

In the spaces x and the time domain t, we adopt a numerical inversion method for the final solution of the temperature distributions. The numerical results have been obtained based on Stehfest [22]. In this method, the inverse $\Omega(x,t)$ of the Laplace transform $\bar{\Omega}(x,s)$ is approximated by the relation

$$\Omega(x,t)=\frac{ln2}{t}\sum_{j=1}^{M}V_j\bar{\Omega}\left(x,j\frac{\ln 2}{t}\right), \tag{14}$$

where V_j is given by the following equation:

$$V_j=(-1)^{\frac{M}{2}+1}\sum_{k=\frac{i+1}{2}}^{min(i,\frac{M}{2})}\frac{k^{\frac{M}{2}+1}(2k)!}{(\frac{M}{2}-k)!k!(i-k)!(2k-1)!}. \tag{15}$$

4. Thermal damage

The assessment of burn is part of the ultimate significant characteristics in the bioengineering science in skin tissues. Accurate prediction of thermal damage for skin tissue is helpful for heat therapy. In order to quantify the thermal damage, one can use the method developed by Henriques and Moritz [23,24]. It can be expressed as follows:

$$\Omega=\int_0^t Be^{-\frac{E_a}{RT}}dt, \tag{16}$$